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Evaluation model of the grey fuzzy on eco-environment vulnerability

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Abstract: The basic theory and evaluation index system of eco-environment vulnerability were reviewed. Based on the grey theory and fuzzy mathematics, a new comprehensive evaluation method from qualitative to quantitative, called grey-fuzzy evaluation, was proposed for evaluating eco-environment vulnerability. It was integrated of Association for Healthcare Philanthropy (AHP), grey correlation analysis, grey statistics and fuzzy judgment. The constitutional principle and method of the new evaluation method were given and its feasibility and effectiveness were proved by the practical example.

Keywords: Eco-environmental vulnerability; Grey theory; Fuzzy mathematics; Comprehensive evaluation

Introduction

The ecological environment is defined as the synthesis of the climate, landform, soil, hydrology, zoology and botany, human activities and so on (Liu 1995). In recent years, the ability of coordination ecosystem oneself is descending continuously and the survival environment for the human is presenting an increasingly weak trend due to the sharp growth of the population and the unreasonable exploitation to the resources. Therefore, the research on the vulnerability of the ecological environment has become a popular issue in the domain of the resource and environment, especially the evaluation of the vulnerability of ecological environment. The object of the research concerning the vulnerability of ecological environment is focused on resource and environment system that is susceptible to human activities and external environment intimidation. The essence of the system, which covers the internal structure and external condition and also reflects the mutual impact between the system of environment resource and the intimidation of the external environment, is described by the vulnerability of ecological environment. The evaluation on vulnerable ecological environment mainly involves the evaluation of instability of the ecosystem owing to the disproportion of various factors in space and time and the impact evaluation of the ecosystem because of human activities and the intimidation of the external environment. The evaluation of eco-environmental vulnerability is a synthesis method concerning the assessment of population, production, social development and the capacity of the whole environment or the degree of capability (Yang 1992). The assessment is the whole evaluation of ecosystem effects that is for the special attributes of the various environmental factors and factors combination, which is known as the scope of eco-environment vulnerability and the evolution tendency. We combined the comprehensive analysis on the reason of frail ecosystem and environment receptor, with the analysis method of fixed-quantity and semi-quantitative, in order to generalize integrant of eco-environment frailty (Li *et al*. 2002). This work not only takes advantage of the quantitative contrast in the regions, also provides the basis for the whole treatment to the vulnerability of ecological environmental synthesis.

The basic theory of grey-fuzzy evaluation on eco-environment vulnerability

Considering the complex system with a typical of the incomplete information, we use the grey system theory to cope with the observational data in some layers by taking the non-statistic mathematical method (a better practicability under the condition less dates and not meeting the request of the system) in order to obtain a better understanding of the interior variety of the system and correlation mechanisms in a higher level (Deng 1990). However, the fuzzy mathematics is one kind of mathematical method that is applied to describe and process the fuzzy information (Qu *et al.* 1988). On the other hand, it brings the mathematic into an objective fuzzy field and sets up a bridge between the traditional classical mathematics and the fuzzy real world. When a fuzzy phenomenon is described by taking this method, its essence and rule are revealed.

The vulnerability of ecological environment has a typical fuzziness such as impact factors (the district of the time and space on the plant), the weight of assessable indexes and the phenomenon and property itself (Liu 1995). In addition, the ecosystem also has the characteristics of the grey system, including the imperfect information, the incomplete relative factors of a ring, the incomplete-known systematic structure and the incomplete understanding of the principle of the system, especially that the distribution between value and time of the ecological environment clearly exhibits a very fuzzy process. Therefore, on the basis of the both characteristics of the ecosystem, this paper builds up a grey fuzzy model of the vulnerability of ecosystem by making use of the method of fuzzy mathematics combined

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with grey system theories.

The evaluation of the grey fuzzy model on eco-environment vulnerability

Grades of vulnerability of ecosystem environment

Ecosystem environment can be divided into 4 grades according to the vulnerable degree of the ecosystem environment. V={the most vulnerable v 1, the more vulnerable v 2, the medium v 3, light vulnerable v 4}.

Establishing the evaluation model with gather hiberarchy

There are control points with the number of "n" in the valuation area. Each control point needs to consider the primary factors with the number of "m" which includes "K_i" (i=1,2,…,m) belonging to the secondary factors. Set up a structure model with three layers: target layer (flimsiness), stipulation layer (the first class evaluation factor), Index layer (the second class evaluation factor). The model structure is shown as Fig. 1.

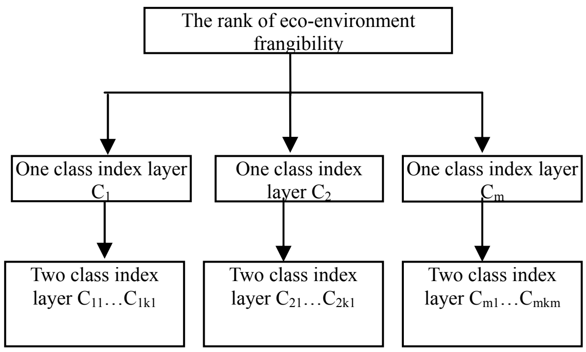

Fig. 1 The structure of evaluation model with gather hiberarchy

Evaluation index system

According to the vulnerable characteristics of ecosystem environment, the vulnerable valuation should be a comprehensive work for each main factor of ecosystem environment in the district. The key of the vulnerable evaluation is to set up a reasonable system of assessable indexes.

The evaluation index system can estimate whether the evaluation result is reliable and possible or not. Because of the vulnerable changes coming from a coefficient difference between natural factors and artificial factors, the index system should be a synthetic system, and the index system chosen should be overall contexts and reflect the actual conditions of natural ecological environment.

Natural factors include architectonic, morphologic characteristics, surficial constructional materials, phytocommunity and biotic population type; Climatic factors mainly include light, hot, water and other factors, which are the material basis for the ecological environment constituents; the conditions and spectrum of economic development are presented by the degree of forest coverage, annual precipitation, potential evapotranspiration, per capita possession of water resources, per capita income, agricultural modernization degree, power farming agricultural acreage as well as chief agricultural acreage, the knowledge population expressed by the ratio between people who have received higher education from universities and technical secondary schools and the total population, and nutrition indexed by the Engel Coefficient.

Quantitative and qualitative indicators are dealt with standardization by using different methods.

The excellent process of the quantitative indexes

The assessable indexes are divided into three types: cost index, efficiency index and zone index. The lower the cost index the better and the higher the efficiency index the better, such as the using rate of resource, satisfaction degree and so on. The zone index means a kind of index which takes a certain fixed zone belonging to, such as advanced index. According to the classification of the indexes, the Q_i also is divided into three subclass set further, even if:

$$Q=\bigcup_{i=1}^{3}Q_i \quad Q_\mathrm{i}\cap Q_j=\varnothing \quad ij=1,2, \quad i\neq j \tag{1}$$

where, The Q_i (because of the inconsistence of quantity key link in the various indexes, the result is not carried on contrast analysis directly, i=1,2,3) represents respectively cost type set, performance type set and zone index set, and the Ø signifies an empty set.

No method can be used directly for comparative analysis due to inconsistent dimensional indicators. Therefore, for different types of superscript, we should adopt different non-dimensional methods (Yang 1992).

The transform equation of non-dimension for cost index is as follows:

$$b_{ij}=\frac{d_i^{\max}-d_{ij}}{d_j^{\max}-d_j^{\min}} \tag{2}$$

i=1,2,…, m; j=1,2,…,n and $O_j \in Q_1$

Where, the $d_j^{\max}$ and the $d_j^{\min}$ represent respectively the maximum and minimum in the allover evaluations

$$d_{ij}=\max\{d_{1j},d_{2j},...,d_{mj}\};\ d_{ij}=\min\{d_{1j},d_{2j},...,d_{mj}\} \tag{3}$$

The transform equation of non-dimension for efficiency index is as follows:

$$b_{ij}=\frac{d_{ij}-d_j^{\max}}{d_j^{\max}-d_j^{\min}} \tag{4}$$

i = 1 , 2 , … , m ; j = 1 , 2 , … , n and $Q_j \in Q_2$

The transform equation of non-dimension for zone index is:

$$b_{ij}=\begin{cases}1-\dfrac{S_1-d_{ij}}{\max\{S_1-d_j^{\min},d_j^{\max}-S_2\}} & d_{ij}<S_1\\ 1 & d_{ij}\in[S_1,S_2]\\ 1-\dfrac{d_{ij}-S_2}{\max\{S_1-d_j^{\min},d_j^{\max}-S_2\}} & d_{ij}>S_2\end{cases} \tag{5}$$

i=1,2,…,m; j=1,2,…,n and $O_i \in Q_2$

where, [S_1, S_2] is the best interval for the O index

Fuzzy quantification qualitative index

The trapezoidal fuzzy number is used to express the indexes, Among these indexes, “bad” corresponds with trapezoidal fuzzy number (0, 0, 0, 0, 2), “a little bad” corresponds with the number (0, 0, 2, 0, 2, 0, 4), “common” corresponds with the number (0, 3, 0, 5, 0, 5, 0, 7), “a little good “corresponds with trapezoidal fuzzy number (0, 6, 0, 8, 0, 1, 0), “good” corresponds with the number (0, 7, 0, 9, 1, 0, 1), and “very good” corresponds with the number (0, 8, 1, 0, 1, 0, 1, 0). For the convenience of calculation, the trapezoidal fuzzy numbers are often written as L-R fuzzy number, and the relationship is: trapezoidal fuzzy numbers (φ, m, n, β), L-R fuzzy numbers (m, n, γ, δ), γ = m- φ, δ = β – n, L-R fuzzy number M = (a, b, φ, β), N= (c, d, γ, δ). According to the literature (Yang 1992), the equation used for the division of fuzzy numbers for the trapezoid fuzzy number can be expressed as:

$$[M / N] \approx \frac{a}{d}; \frac{b}{c}; \frac{a\delta + d\varphi}{d(d+\delta)}; \frac{b\gamma + c\beta}{c(c-\gamma)}] \tag{6}$$

The overall expectations for the trapezoid fuzzy number (Ã = (φ, m, n, β))) are:

$$I_{\bar{A}} = \left(\frac{\alpha + m + n + \beta}{\Delta} \right) \tag{7}$$

Fuzzy grey correlation analysis

The original time series must be dimensionless. To avoid the impact of inconsistent unit to correlation analysis, the starting of the various time series should be coincident after treatment, and the new time series were respectively recorded as $x_0^{(0)}$ (t) and $x_i^{(0)}$ (t) .

$$x_0^{(0)}(t)=\{x_0^{(0)}(1), x_0^{(0)}(2),\ldots, x_0^{(0)}(m)\}= \left\{ \frac{x_0(1)}{x_0(1)}, \frac{x_0(2)}{x_0(1)}, \ldots, \frac{x_0(m)}{x_0(1)} \right\} \tag{8}$$

$$x_i^{(0)}(t)=\{x_i^{(0)}(1), x_i^{(0)}(2),\ldots, x_i^{(0)}(m)\}= \left\{ \frac{x_i(1)}{x_i(1)}, \frac{x_i(2)}{x_i(1)}, \ldots, \frac{x_i(m)}{x_i(1)} \right\} \tag{9}$$

In order to move comparative time sequence toward the reference sequence, a linear transform should be done, which can make the starting and ending value coincide with the reference sequence without the change of curve shape. The new sequence transformed is called generating sequence, $X_i^{(1)}(t)$,

$$X_i^{(1)}(t) = \omega + \frac{x_i^{(0)}(t) - 1}{x_i^{(0)}(m) - 1}(\omega - \theta) \tag{10}$$

Where, (ω, θ) is a range for generating sequence, generally [1, $x_0^{(0)}(m)$].

If *T* represents time, the difference of absolute value between $x_0^{(0)}$(t) and $x_i^{(1)}$(t) can be expressed as $\Delta_{oit}=| x_0^{(0)}(t) - x_i^{(1)}(t) |$, t=1,2,…,m; i=1,2,…,n. According to the expression, the minimum difference of absolute value can be expressed as $\Delta_{\min} = \min_i \min_t |X_0^{(0)}(t) - X_i^{(1)}(t)|$ and the maximum difference of absolute value for the moment can be expressed as $\Delta_{\max} = \max_i \max_t |X_0^{(0)}(t) - X_i^{(1)}(t)|$. Generally, the distinguishing coefficient is 0.5. Actually, the correlation coefficient ξ oi(t) is variable and correlated with X_i and other comparative sequences X_h (h=1,2,…,n). Because the maximum absolute value can reveal the whole characteristic of the system, the distinguishing coefficient D as a maximum should reflect sufficiently the indirect impact of various factors to associated space and play a partial role in resisting interference. Namely it can weaken the error generated from the abnormal observation of the entire associated space.

The method of defining the distinguishing coefficient D is as follows (Li *et al*. 2002; Koczy 1983)

$$\Delta_k = \frac{1}{5m} \sum_{i=1}^{5} \sum_{i=1}^{m} | X_0^{(0)}(t) - X_i^{(0)}(t)| \tag{11}$$

If ŋ=$\Delta_k/\Delta_{\max}$, the range of δ is ŋ≤δ≤2 which should meet the conditions that $\Delta_{\max}>3_{\Delta k}$, ŋ≤δ≤1.5 ŋ; or $\Delta_{\max}>3_{\Delta k}$, 1.5 ŋ<δ≤2 ŋ

After ascertaining the distinguishing coefficient, the correlation coefficient can be calculated by the Eq. (12), in order to focus on the whole correlation space:

$$E_{0i}(t) = \frac{\max_i \max_t |x_0(t) - x_i(t)|}{|x_0(t) - x_i(t)| + \delta \max_i \max_t |x_0(t) - x_i(t)|} \tag{12}$$

The structure of fuzzy judgment matrix

Based on hierarchical model, the pair factors in the same layer are used in a contrast computing involved the above so as to constitute a contrast judgment Matrix in the both factors (Zhao 1986).

If u_i (i=1,2,…,m) is expressed as the first-grade evaluation factor, A—C (judgment Matrix) of the first-grade evaluation factor is obtained.

$$Pc = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mm} \end{bmatrix} \tag{13}$$

R_{ij}= (i, j=1,2,…,m) represents the relative important value from u_i to u_j in the first class evaluation factor Pc_i;

Suppose that u_i represents the second-grade evaluation factor, C_i—C_{iai} (ai=1,2,…,K_i) the judgment Matrix *Pc* can be obtained in the same way.

$$Pc_i = \begin{bmatrix} r'_{11} & r'_{12} & \cdots & r'_{1ki} \\ r_{21} & r_{22} & \cdots & r'_{2ki} \\ \vdots & \vdots & & \vdots \\ r'_{ki1} & r'_{ki2} & \cdots & r'_{kiki} \end{bmatrix} \tag{14}$$

Ascertaining the weight of each element of assessment

The eigenvector is calculated by judgment matrix, which is corresponding to the maximum eigenvalue and is the weight value of relative importance

Calculate the product per row element M_i in judgment matrix.

$$M_i = \prod_{j=1}^{m} r_{ij}, (i, j = 1,2,...,m); \mathrm{W_i} = \frac{W_i}{\sum_{j=1}^{m} W_j} \tag{15}$$

Calculation of W_i, it is expressed by $\sqrt[m]{M_i}$, $W_i = M_i^{\frac{1}{m}}$; Normalization of the vector W:

$W=[W_1,W_2,\ldots,W_m]^T$, so $W=[W_1,W_2,\ldots,W_m]^T$ is the required eigenvector used to calculate the maximum estimate $\lambda_{\max}$ in judgment matrix

$$\lambda_{\max} = \sum_{i=1}^{m} \frac{(PW)_i}{nW_i} = \frac{1}{n} \sum \frac{(PW)_i}{W_i} \tag{16}$$

Where, $(PW)_i$ is the i element of the vector PW.

$$PW = \begin{bmatrix} (PW)_1 \\ (PW)_2 \\ \vdots \\ (PW)_m \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \vdots & \vdots & & \vdots \\ u_{m1} & u_{m2} & \cdots & u_{mm} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_m \end{bmatrix} \tag{17}$$

The test of the weight: since the above eigenvector is the required weight, it is necessary to proceed consistency check for the judgment matrix, in order to prove the rationality.

$$CR = \frac{CI}{RI} \;;\; CI = \frac{1}{m-1} C(\lambda_{\max} - m) \tag{18}$$

where, CR is the random consistency ratio of the judgment matrix, CI the common coincidence indicator of the judgment matrix, and RI the average random coincidence indicator of the judgment matrix for the judgment matrix 1 – 9. The value of the RI could refer to the literature (Zhao 1986).

When CR<0.10, the content consistency of judgment matrix is reliable, and also indicates that the weight distribution of the evaluation factor is reasonable; otherwise, the judgment matrix should be adjusted until the content consistency. The eigenvector of the first-grade evaluation factor in judgment matrix can be acquired by these steps, and the eigenvector is the weight multitude $Ac = (W_1, W_2, \ldots, W\text{m})$ of the first-grade evaluation factor. In the same way, the eigenvector of the second-grade evaluation factor in judgment matrix can also be acquired. The eigenvector is $W_{ci}=(W_{i1},W_{i2}\ldots,W_{iKi})\mathrm{T}$. According to $W_{ci}=W_i \cdot W_{ij}$ (i=1,2,…,m; j=1,2,…,K_i), we could establish the weight multitude, $A_{ci}=W_i \cdot W_{ci}=(W_{ci1},W_{ci2},\ldots,W_{ciki})$, of the second-grade evaluation factor.

Establishing fuzzy evaluation matrix for each layer

The fuzzy evaluation matrix with the single-factor evaluation can be used as an assessment of various grades of vulnerability evaluation.

The index information of every single-factor in the ecological environment is limited. As far as the whole process, it is born with the characteristic of the grey system. Thus, the rating of grey (grey-fuzzy membership) to each single factor in the various grades of evaluation is determined by the grey statistical analysis of single-factor. The fuzzy relationship between the constitutional factor being composed of the grey-fuzzy membership and grades of vulnerability is defined as the evaluation matrix (σ_i). The grey-fuzzy membership of eco-environment vulnerability scale can be calculated by grey statistical analysis.

And Ⅰ, Ⅱ, Ⅲ,...,ω: statistical analysis; 1° and 2°,3°...,m°: unfavorable for statistical indicators; 1,2,3,...,n: statistical grey; f_1, f_2,...,f_n: grey category weighted functions; Real number d_{ij} is the object (sample) which is the i statistical to the j indicators data. d_{ij}, $i \in$ {Ⅰ, Ⅱ,...,ω}, $j \in$ {1°,2°,…,m°}, D is the matrix with elements in d_{ij} (Turksen *et al*. 1994).

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ d_{\omega 1} & d_{\omega 2} & \cdots & d_{\omega m} \end{bmatrix} \begin{matrix} \mathrm{I} \\ \mathrm{II} \\ \vdots \\ \omega \end{matrix} \tag{19}$$

If F is mapping, $O_p f_k(d_{ij})$ is a calculation of $f_k(d_{ij})$, and σ_i is the Weights (sequence): grey ash 1, grey ash 2,…, grey ash n. $\sigma_j=(\sigma_{j1}, \sigma_{j2},\ldots,\sigma_{jn})$, $j \in$ {1°,2 °,…,m° }. When F: $o_p f_k(d_{ij}) \to \sigma_{jk} \in [0,1]$, $K \in$ {1,2,…,n}, $i \in$ {Ⅰ, Ⅱ,…,ω}, $j \in$ {1°,2°,…,m°}, $F(o_p f_k(d_{ij}))$ is the grey statistics that all statistical indicators for the j target.

$$F\left(O_P f_k\left(d_{ij}\right)\right) = \sum_{i=1}^{\omega} f_k d_{ij} \Big/ \sum_{k=1}^{n} \sum_{i=1}^{\omega} f_k(d_{ij}) \sigma_{jk}, \tag{20}$$

$f_k(d_{ij})$ satisfy: grey $\otimes \in [x_2, \infty]$;

$$\begin{cases} f_k(d_{ij}) = L_k(d_{ij}) \dfrac{d_{ij} - x_1}{x_2 - x_1}, & d_{ij} \in [x_1, x_2], \\ f_k(d_{ij}) = 1, & d_{ij} \in (x_2 \infty), \end{cases} \tag{21}$$

grey $\otimes \in [x_1, x_2, x_3]$;

$$
\begin{cases}
f_k(d_{ij}) = L_k(d_{ij}) = \dfrac{d_{ij} - x_1}{x_2 - x_1}, & d_{ij} \in [x_1, x_2], \\
f_k(d_{ij}) = 1, & d_{ij} = x_2 \\
f_k(d_{ij}) = R_k(d_{ij}) = \dfrac{x_3 - d_{ij}}{x_3 - x_2}, & d_{ij} \in [x_2, x_3], \\
f_k(d_{ij}) = 0, & d_{ij} \in [x_1, x_3]
\end{cases} \tag{22}
$$

grey $\otimes \in [0, x_1, x_2]$;

$$
\begin{cases}
f_k(d_{ij}) = 1, & d_{ij} \in [0, x_1] \\
f_k(d_{ij}) = R_k(d_{ij}) = \dfrac{x_2 - d_{ij}}{x_2 - x_1}, & d_{ij} \in [x_1, x_2]
\end{cases} \tag{23}
$$

sign: $\otimes \in [x_2, \infty]$; $\in \otimes [x_1, x_2, x_3]$; $\in \otimes [x_1, x_2, x_3]$, as shown in Fig. 2, 3, 4, respectively.

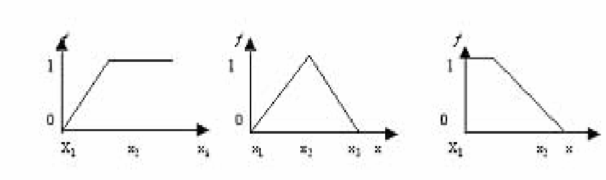

Fig.2 $\in \otimes (x_2, \infty)$ Fig.3 $\in \otimes (x_1, x_2, x_3)$ Fig.4 $\in \otimes (x_1, x_2, x_3)$

$$
\sigma_j = (\sigma_{j1}, \sigma_{j2}, \ldots, \sigma_{jn}) = \left[\frac{\sum_{i=1}^{\omega} f_1(d_{ij})}{\sum_{k=1}^{n} \sum_{i=1}^{\omega} f_k(d_{ij})}, \frac{\sum_{i=1}^{\omega} f_2(d_{ij})}{\sum_{k=1}^{n} \sum_{i=1}^{\omega} f_k(d_{ij})}, \ldots, \frac{\sum_{i=1}^{\omega} f_n(d_{ij})}{\sum_{i=1}^{n} \sum_{j=1}^{\omega} f_k(d_{ij})} \right] \tag{24}
$$

Evaluation matrix (σ_i) is defined as the fuzzy relationship between the grey-fuzzy membership and vulnerability scale, namely constituted fuzzy evaluation matrix of the first-grade factor.

$$
\mathrm{Rci} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1\mathrm{ki}} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2\mathrm{ki}} \\ \vdots & \vdots & \cdots & \vdots \\ \sigma_{\mathrm{ki}2} & \sigma_{\mathrm{ki}2} & \cdots & \sigma_{\mathrm{kiki}} \end{bmatrix} \tag{25}
$$

where σ_{ij} (i,j=1,2,…,k_i) is the grey-fuzzy membership, which is the first evaluation u_i to the weights of u_j

Evaluation models on the grey-fuzzy

The evaluation model of the grey fuzzy is a mathematics tool which the synthesis determination is implemented by adopting fuzzy mapping and the fuzzy linear substitution. According to the numbers of factors involved in the judgment, the fuzzy comprehensive evaluation is divided into the single fuzzy synthetic evaluation and the multistage fuzzy synthetic evaluation. In addition, the synthetic evaluation is also divided into two levels. The first level is the A_{Ci}, (the weight assignment of evaluation factor in the second level that is determined by the hierarchy analytic process) and the R_{Ci} (the evaluation matrix determined by the first level) carries out synthesis operation in order to come out the judgment of the level factor, which is called the first level fuzzy synthetic evaluation model:

$$
\begin{aligned}
\mathrm{B}_{\mathrm{ci}} &= \mathrm{A}_{\mathrm{Ci}} \bullet \mathrm{R}_{\mathrm{Ci}} \\
&= (\mathrm{W}_{\mathrm{Ci1}}, \mathrm{W}_{\mathrm{Ci2}}, \cdots, \mathrm{W}_{\mathrm{CiK}}) \bullet \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1\mathrm{ki}} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2\mathrm{ki}} \\ \vdots & \vdots & \cdots & \vdots \\ \sigma_{\mathrm{ki}2} & \sigma_{\mathrm{ki}2} & \cdots & \sigma_{\mathrm{kiki}} \end{bmatrix} \\
&= (\mathrm{B}_{\mathrm{Ci1}}, \mathrm{B}_{\mathrm{Ci2}}, \cdots, \mathrm{B}_{\mathrm{CiKi}})
\end{aligned} \tag{26}
$$

The second level fuzzy synthetic evaluation models：

$A_C = (W_1, W_2, \ldots, W_m); B = A_i \bullet R_i = (W_1, W_2, \ldots, W_m) \bullet R_i$

The process is that $B_1, B_2, \ldots, B_m$ (the result of the first level evaluation in the second level evaluation factors) is arranged in longitudinal row to constitute second level evaluation matrix .

The overall merit model for the second-grade fuzzy is showed as follows:

$$
B = A_i \bullet R_i = (W_1, W_2, \cdots W_m) \bullet R_i
$$

$$
= (W_1, W_2, \cdots W_m) \bullet \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix} = (v_1, v_2, v_3, v_4)
$$

（27）

On the basic principle of the maximum grade in fuzzy mathematics, the vulnerable scale of ecological environment can be evaluated to some extent.

Examples

Wuchuan County, Inner Mongolia is located in the south of Inner Mongolia plateau, north foot of Yinshan Mountain. It has a typical temperate, semi-arid continental monsoon climate. The annual average temperature is 3.1 °C and the annual average rainfall is about 300 mm, mainly concentrating in July to September. The main vegetation type is dry grassland, shrub and desert plant, and the main agricultural crops are wheat, oat, etc. The economy of agriculture and animal husbandry is backward, and desertification and soil erosion are increasingly serious in this region.

Identification of the hierarchical structure and evaluation indexes

The eco-environment of the staggered areas between agriculture and animal husbandry in the north of China is very complex, which the cause involves many factors and is complex. The key is to choose evaluation factors. In this paper, 13 indicators were collected according to the principle of desirability, concise and comparability.

The fuzzy matrix

The fragile indexes of eco-environment at Wuchuan County were calculated by the grey correlation, establishing the fuzzy judgment matrix.

Based on the calculation of the matrix eigenvalue, eigenvalue vector and consistency test, the distribution of the total weight and subsystems weight can be obtained. A (Total weight)=(0.66, 0.15, 0.09, 0.10); A1(natural resources subsystem weight)=(0.68, 0.23, 0.09); A2(environment subsystem weight)=(0.61, 0.25, 0.08, 0.06); A3 (economy subsystem weight)=(0.77, 0.16, 0.07); A4 (social development subsystem weight)=(0.76, 0.12, 0.12).

Fuzzy evaluation matrix

Each evaluation grey weight of various factors was obtained by analyzing grey single-factor, and grey membership category. Evaluation matrix was defined as the fuzzy between the component factors and vulnerability grades. The results were shown as follows:

Natural subsystem evaluation matrix R_1

$$R_1 = \begin{bmatrix} 0.57 & 0.22 & 0.18 & 0.03 \\ 0.43 & 0.41 & 0.07 & 0.07 \\ 0.48 & 0.16 & 0.24 & 0.12 \end{bmatrix}$$

Environmental subsystem evaluation matrix R_2

$$R_2 = \begin{bmatrix} 0.68 & 0.24 & 0.04 & 0.04 \\ 0.46 & 0.32 & 0.14 & 0.08 \\ 0.38 & 0.40 & 0.16 & 0.06 \\ 0.39 & 0.36 & 0.13 & 0.12 \end{bmatrix}$$

Economic subsystem evaluation matrix R_3

$$R_3 = \begin{bmatrix} 0.46 & 0.38 & 0.09 & 0.07 \\ 0.44 & 0.32 & 0.16 & 0.12 \\ 0.38 & 0.36 & 0.10 & 0.16 \end{bmatrix}$$

Social development subsystem evaluation matrix R_4

$$R_4 = \begin{bmatrix} 0.32 & 0.48 & 0.12 & 0.08 \\ 0.26 & 0.28 & 0.22 & 0.24 \\ 0.28 & 0.40 & 0.20 & 0.12 \end{bmatrix}$$

Respectively bring the evaluation matrix and weight into the grey fuzzy evaluation models, we can get: B= (0.54 0.19 0.19 0.08). According to the principle of largest membership in fuzzy mathematics, we can conclude that the ecological environment at Wuchuan County, Inner Mongolia belongs to the extremely fragile district.

Conclusions

The evaluation model of eco-environment vulnerability which uses grey theories and vague mathematics gathers many experts' idea. The model carries forward the advantages of the classic method, also controls and makes up its partial weaknesses such as people's mistakes. The practice confirmed that this method was effective and reliable for solving vulnerability evaluation of eco-environment. Moreover, it also provides an important basis for solving the similar evaluation problems, especially involving the fuzzy and multi-level grey-fuzzy analysis.

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